RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College under University of Calcutta)

FIRST YEAR

B.A./B.SC. FIRST SEMESTER (July – December), 2011

Mid-Semester Examination, September, 2011

Date : 15/09/2011

MATHEMATICS (General)

Time : 11 am – 12 noon

Paper : I

Full Marks : 50

(Use separate answer scripts for each group)

Group - A

1. Answer **any three** questions : [3×3 = 9]

a) Find the general value of i^i. [3]

b) State De Moivre's theorem. Using De Moivre's theorem express $\cos 3\theta$ in terms of powers of $\cos\theta$ where θ is real. [1+2]

c) If n is a positive integer and α, β are roots of $x^2 - 2x + 2 = 0$ then using De Moivre's theorem show that $\alpha^n + \beta^n = 2^{\frac{n}{2}+1} \cos\frac{n\pi}{4}$. [3]

d) Find the value of $\sqrt{-3+\sqrt{-3+\sqrt{-3+\ldots}}}$ and also find Log i. [2+1]

e) If $Z_r = \cos\frac{\pi}{3^r} + i\sin\frac{\pi}{3^r}$ $(r = 1, 2, \ldots)$ then prove that $Z_1 Z_2 Z_3 \ldots \infty = i$ [3]

Group – B

2. Answer **any two** questions : [2×4 = 8]

a) Define Cartesian product of two sets. If A = {1, 2, 3}, B = {2, 4, 6} then find A×B. Find also P(A), the power set of A. [1+1+2]

b) Prove that the composition of two injective maps is injective. Verify injectivity and surjectivity of $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = K^2 + 1 \; \forall \; n \in \mathbb{R}$ [2+2]

c) Define the eigen-value of a square matrix. Find the eigen-values of $A = \begin{pmatrix} 1 & -1 & 0 \\ 1 & 2 & -1 \\ 3 & 2 & -2 \end{pmatrix}$ [1+3]

Group – C

Answer **any two** questions : [2×4 = 8]

3. a) Define the continuity of a function. Discuss the continuity of the function

$$f(x) = \begin{cases} x^2 \sin\frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

at x = 0

Give an example of a discontinuous function. [1+2+1]

b) A function which is finitely derivable at a point is continuous at that point. Is the converse true? Give reason. [2+2]

c) If f(x+y) = f(x)f(y) for all real values of x, y; $f(x) \neq 0$ for any real value of x and f´(0) = 2, Prove that f´(x) = 2f(x). [4]